IPB 0121

NOISE PERFORMANCE OF VECTOR SUM LOCK-IN AMPLIFIERS

DL INSTRUMENTS

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With the growing popularity of phase–independent* lock–in amplifiers, there has been some confusion regarding the attainable signal–to–noise ratio of vector sum lock–in amplifiers. From a cursory inspection it might appear that a vector sum output...

$$A = \sqrt{X^2 + Y^2}$$

where...

$$X = A\cos\phi$$
$$Y = A\sin\phi$$

is necessarily noisier than either of its inputs. This is not true, as will be shown.

For a given set of input signal conditions, the output signal–to–noise ratio of a true vector sum lock–in is essentially the same as that of a single phase lock–in with the output maximized by use of the phase controls.

The block diagram of a modern Vector Sum Lock–In Amplifier (ITHACO DYNATRAC® Model 393) is shown in Figure 1 below.

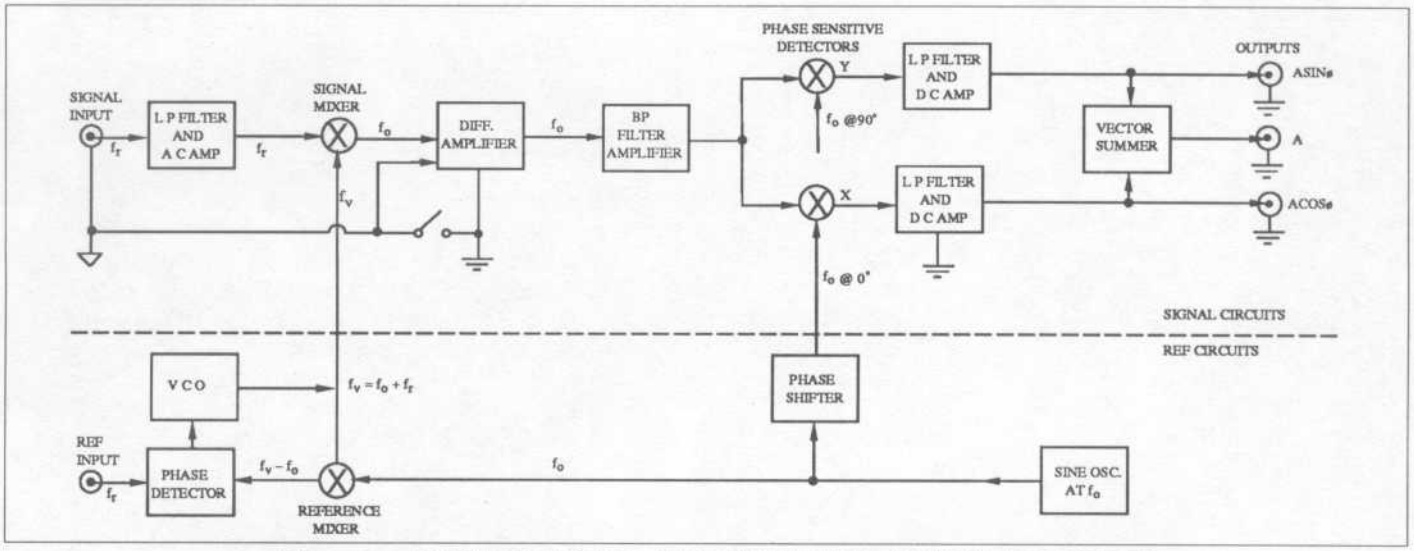

Figure 1 BLOCK DIAGRAM – DYNATRAC® 393 VECTOR SUM LOCK–IN

THEORY:

A two phase lock–in with noise only at its input will have outputs with normal (Gaussian) distribution and zero mean. If the noise on each output has rms value σ the probability density of each is:

$$p(E) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{E^2}{2\sigma^2}\right) \quad (1)$$

By vector summing the two uncorrelated noise voltages ($e_v = \sqrt{e_1^2 + e_2^2}$) one obtains on the output of the vector summer a voltage that is Raileigh distributed. The Raileigh density function is:

$$p(E) = \frac{E}{\sigma^2} \exp\left(-\frac{E^2}{2\sigma^2}\right) \quad (2)$$

If a sinusoid with amplitude A is added to the noise, the vector summer output has the density function :

$$p(E) = \frac{E}{\sigma^2} \exp\left(-\frac{(E^2 + A^2)}{2\sigma^2}\right) I_0\left(\frac{EA}{\sigma^2}\right) \quad (3)$$

(I is the modified Bessel function of the first kind, here of order zero.)

For large A/σ values the density approaches:

$$p(E) \simeq \frac{1}{\sigma}\sqrt{\frac{E}{2\pi A}} \exp\left(-\frac{(E-A)^2}{2\sigma^2}\right) \quad (4)$$

This approximates a normal distribution centered around E = A with standard deviation σ. But that is exactly what one would obtain with a single channel lock–in with the proper phasing.

*It is important to distinguish between a true vector sum output and the "vector amplitude" mode of certain older two phase lock–in's.

The technique used in these older instruments involves feedback from the quadrature (Asinø) output to a voltage controlled phase shifter which nulls that output and peaks the in–phase (Acosø) output. Because of the feedback, the effective bandwidth is substantially increased. In turn, this results in susceptibility to noise and interference that is much higher than in the normal mode of operation. The phase tracking range of this technique is also limited.

One way of explaining this is to assume that the two phase lock–in is phased so that the in–phase output voltage is maximized. With large A/σ, that voltage would be much larger than the voltage from the quadrature channel so as to completely control the vector summer output:

$$\sqrt{e^2_1 + e^2_2} \sim e_1 \quad \text{if } e_1 \gg e_2$$

In other words, the vector summer output is nearly identical to the in–phase output. Thus it must have nearly the same dc component and noise component and therefore the same signal–to–noise ratio.

The actual dc component of the vector summer is slightly higher. The mean of the voltage expressed in equation (3) can be calculated to be:

$$\overline{E} = \sqrt{\frac{\pi}{2}}\,\sigma \exp\left(\frac{A^2}{4\sigma^2}\right)\left[\left(1+\frac{A^2}{2\sigma^2}\right)I_0\left(\frac{A^2}{4\sigma^2}\right) + \frac{A^2}{2\sigma^2}I_1\left(\frac{A^2}{4\sigma^2}\right)\right] \quad (5)$$

Using power series expansions of the Bessel functions we obtain after lengthy calculations:

$$\overline{E} = A\left(1 + \frac{1}{2\left(\frac{A}{\sigma}\right)^2} + \frac{1}{8\left(\frac{A}{\sigma}\right)^4} + \frac{3}{16\left(\frac{A}{\sigma}\right)^6} + \ldots\right) \quad (6)$$

Showing that $\overline{E} \rightarrow A$ for large A/σ.

Equation (6) may be used for $A/\sigma \geq 2$ with error less than .1%. The table below shows E/A for various A/σ values.

$A/\sigma = \frac{\text{Output Signal}}{\text{rms Noise}}$	$E/A = \frac{\text{Actual dc out}}{\text{dc Out with No Noise}}$
2.0	1.136
2.5	1.084
3.0	1.057
3.5	1.042
4.0	1.032
5.0	1.020
6.0	1.014
8.0	1.008
10.0	1.005
15.0	1.002
20.0	1.001
40.0	1.000

What this table says, in effect, is that if the time constant switch is set for a noise level at the output that is less than 10% of the dc signal at the output, the measurement is less than .5%.

EXPERIMENTAL VERIFICATION:

To illustrate the operation of the vector summer on a noisy signal, a Model 393 Lock–In Amplifier was connected to a signal source comprising a variable noise voltage and a variable 1 kHz sine voltage.

The noise voltage was adjusted for a σ = .5V rms output voltage on the Acosø output (measured with a Ballantine RMS Meter). For each A value from 5V to zero the noise source was switched on briefly to show the effect of the noise. The A and Acosø outputs were recorded on a chart recorder and are shown in Figure 2.

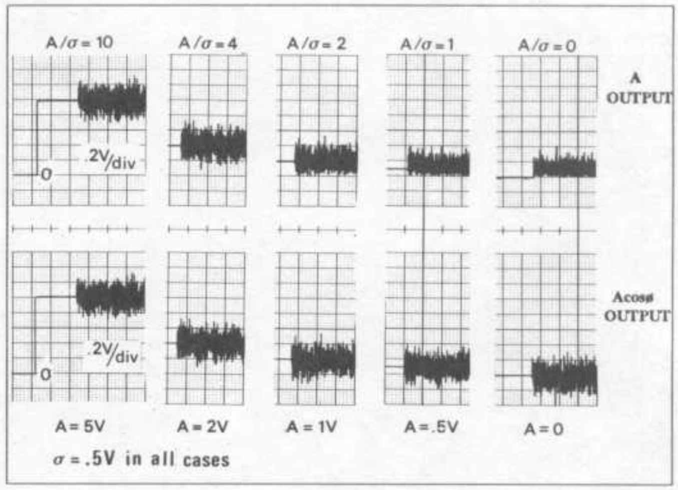

Figure 2

Note that for A = 5V, the output signal–to–noise ratio is A/σ = 10. The error in the average value of the noise output from the vector summer (A output) should be 25mV or one eighth of the smallest division on the chart recording. This small error is certainly not visible in the several volts swing of the output!

Figure 3 shows A and Acosø outputs as the phase shifter is varied over 360°. Note that the vector sum output A does not change.

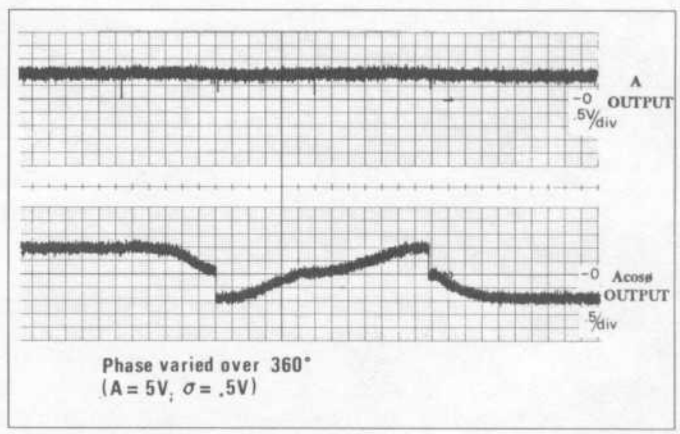

Figure 3

CONCLUSION

The conclusion must be that when measuring the amplitude of synchronous signals with a useable output signal–to–noise ratio, the vector summer does not affect the performance of the lock–in.

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